

Princeton University, Bept. of Elec. Eng. & computer secure

Final Report

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Activity under this contract has centered on two general problem areas related to the airborne detection of underwater targets. The first utilizes anomalies in the earth's magnetic field to detect the presence of a target, while the second is a detection problem associated with returns from an optical radar.

Two models and approaches for the detection of targets by means of measuring magnetic anomilies have been studied. In the first, the nonlinear equations relating target position to it's magnetic signal are used directly. After a preliminary study of some recursive estimation techniques, a more detailed study of using an extended Kalman filter as an off-line sequential estimator was undertaken. A novel idea in this approach was to process the data with random samples rather than in the causal sequential order in which they occur. As discussed below in Section A.1, this enhances convergence of the parameter estimates since it avoids directly processing correlated data.

The second model is a linear version of the nonlinear equations, and a combined detection - estimation problem is formulated. As discussed in section A.2, approximate non-Gaussian (and robust) state estimation combined with sequential likelihood ratio processing appears to be an attractive approach for detecting targets in a *switching environment,* and also where models are not very refined.

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In the second general problem area, optical radar returns are modeled as point processes. For interarrival time observations, optimal (likelihood ratio) and suboptimal detector systems have been investigated. Focus has been directed on a canonical receiver structure and effective adaptive (or learning) procedures. Adaptive procedures are particularly important in this instance since the channel model (the ocean-air medium) is not very well known. This is discussed further in section B below.

A. Recursive Procedures for Magnetic Detection of Targets

We have investigated two models and approaches for the detection and tracking of underwater targets through magnetic anomaly observations. The first approach is a combined detection - estimation problem with a linear model. In the second, the nonlinear equations which represent the target's induced magnetic field are dealt with directly.

A.1 Recursive Nonlinear Parameter Estimation

A magnetic dipole is induced in a steel target, which results in an anomaly of the earth's magnetic field. At distances several times greater than the target's length, the field intensity is given by ([1], p. 247, Eq. (21)),

$$H = \frac{-M}{4\pi r^3} \left[(\underline{a}_{M} \cdot \underline{a}_{e}) - 3 (\underline{a}_{M} \cdot \underline{a}_{r}) (\underline{a}_{r} \cdot \underline{a}_{e}) \right]$$

where M is the magnetic dipole moment, or the distance between the magnetometer and target, and $\underline{a}_{r} \cdot \underline{a}_{m} \cdot \underline{a}_{p}$ represent unit vectors

in the directions of the distance, magnetic dipole and earth's magnetic field. This equation can be translated into Cartesian coordinates or any other convenient set ([1], Eq. 6). A linearized version is the model to be discussed in the next section. Here, target parameters are estimated directly from the nonlinear equations.

For illustrative purposes, assume there are three parameters, θ_{i} , i=1,2,3, which describe the target. A noiseless magnetometer reading is given by

$$H_{t} = f(\underline{\theta}, t) \tag{2}$$

where t denotes the sampling time and $\underline{\theta}$ represents the vector of the three parameters. With \overline{H}_t the actual reading, the residual term is

$$\begin{array}{ccc}
T & 2 \\
\Sigma & (\overline{H}_t - f(\underline{\theta}, t)) \\
t = 1
\end{array}$$
(3)

One then looks for estimates of $\underline{\theta}$ which minimizes (3) or some related error function. Clearly, it is most desirable to do this recursively and in real time. However, the straightforward application of a recursive estimator to nonlinear regression (curve fitting) problems is sometimes not possible when the sequentially generated data tends to be correlated, i.e., when minimizing the error using a local segment of the data (say the first 100 points) does not ensure that the error is minimized globally. This is

especially true when good a prior parameter estimates are not available.

We have found some of these problems to arise with the above model and using estimation techniques such as multidimensional stochastic approximation and extended Kalman filtering. Some of our recent work under the contract ([2]) illustrates clearly a potential problem: if the error surface is plotted (as a function of two parameters) with the number of observations changing from figure to figure, the multipeak nature of the surface is evident. What this means is that, as discussed above, when using the first 100 data points, there is a tendency to head for local minima. It is only when all the data is processed can you attempt to guarantee a global optimum solution.

There are three alternate possibilities. One is to change the model as discussed in section A.2 below. The second approach is to run parallel sequential estimators on the same data, using different starting points with, for example, a majority test to specify the global optimum point. The third is to process the data offline in a quick and efficient manner. As part of the project activity, we have also investigated this third approach and the results look promising.

Rather than use a conventional iterative technique (i.e., gradient) to process all the data, we have used an extended Kalman filter as an off-line sequential estimator. The reason for this has to do with the potential computational savings. The novel idea

in this approach is to process the measurements in a random order rather than in the causal sequential order in which they occur. This avoids the problem of having to deal with, say, the first 100 observations which tend to be more correlated and lead to parameter estimates which do not approach the true parameters.

We have shown in Ref. 2 that the extended Kalman filter can be viewed as an approximate recursive descent procedure. Using an idea developed in [3], "ficticious" measurement noise is included in a simple manner to enhance convergence. The results of the analysis and computer simulations are given in Ref. 2. An intuitive reason why faster convergence is obtained is because by random sampling over all the waveform (observations) one builds up more quickly a "global surface" and does not initially deal with correlated data.

Since the RSKF (random sampling extended Kalman filter) does not explicitly depend on the particular model, it is potentially as useful in other comparable non-linear regression problems.

A.2 Combined Estimation-Detection

As discussed above, another approach is to work with a different model. If sampling rates are high enough a linear model for air-craft dynamics and magnetometer readings can be used. These equations are:

$$\underline{x}_{k} = A_{k-1}\underline{x}_{k-1} + B_{k-1}\underline{u}_{k-1} + v_{k-1}$$

$$\underline{y}_{k} = H_{k}\underline{x}_{k} + \underline{w}_{k} + \alpha_{k}\underline{z}_{k}$$

where the first equation denotes aircraft dynamics and $\underline{\mathbf{y}}_k$ is the

magnetometer observation vector. The quantities are defined as follows:

 \mathbf{x}_{k} - a vector of six components, denoting aircraft position and velocity.

u, - pilot control inputs.

 v_k - system noise, e.g., wind gusts.

Ak, Bk - system matrices which are time varying, since they represent a linearized model of the aircraft trajectory and, hence, depend on the state of the system.

 \mathbf{y}_{k} - observation vector which is the total magnetic signal recorded by the magnetometer.

w_k - observation and instrument noise. This term includes, for example, ocean swells, geological deposits, geomagnetic influence, and the induced magnetic field of the aircraft.

 H_k - observation matrix which is time-varying for the same reason A_k , B_k are time-varying.

 $\mathbf{z}_{\mathbf{k}}$ - target signal which is modeled as a random vector.

The parameter α_k is a sequence of zeros or ones, depending on whether a target is present or not. (It may or may not be considered a random variable.)

Given the sequence of observations, \underline{y}_k , k=1, 2, ..., k, the detection problem is to decide between the two hypotheses

Ho: no target present, i.e., & =o, all k

 H_1 : target present, not all $a_k = 0$.

The estimation part of the problem is to determine the state, \underline{x}_k of the aircraft. This is needed for at least two reasons. First, the earth's magnetic field is so much larger than that induced by a possible target, it must be subtracted from the observation, i.e., decisions are based on $(\underline{y}_k - \underline{y}_k) = \underline{y}_k - \underline{h}_k \underline{x}_k$. Secondly, \underline{h}_k depends on \underline{x}_k , since, in the linearized model, it represents the gradient of the earth's magnetic field evaluated at the aircraft position.

Project activities concerning this model are presented in Ref. [4]. The problem has been formulated as one of sequential detection (i.e., real-time on-line) in a switching environment (α_k is either 0 or 1, thus introducing another noise-like signal). The Bayesian optimal detector is not practical, since it requires exponentially growing memory and numerical integration of probability density functions. Three suboptimal approaches were investigated:

- i) the decision directed procedure
- ii) the linear least-mean-square-error procedure
- iii) approximate non-Gaussian procedure.

As discussed in [4], these three approaches refer to how the means and variances for the state of the system are computed. A preliminary computer simulation study has indicated that the approximate non-Gaussian approach had the best performance while the decision-directed technique the least favorable. In terms of probability of detection error, in contrast to state estimation error variance, ii) and iii) lead to almost the same results. The

most promising approach is iii) with robust state estimates.

This is so because good models for unwanted signals such as geological deposits, ocean swells, etc..., do not exist.

A.3 Models with Interrupted Observations

We have also investigated an observation equation of the form

$$y_k = y_k H_k x_k + w_k$$

where \mathbf{Y}_k is a multiplicative disturbance, $\underline{\mathbf{w}}_k$ an additive noise, and $\underline{\mathbf{x}}_k$ the information bearing signal. This signal is assumed to evolve through a linear system

$$\underline{x}_{k+1} = A_k \underline{x}_k + \underline{\mu}_k$$

where the $\underline{\mu}_k$ sequence is independent, thus making the signal Markov, a model often used for signal sources.

We studied and answered the following question: can recursive linear estimates of \mathbf{x}_k be obtained? It turns out that estimates of the desired form

$$\underline{\underline{A}}_{k+1} = F_k \underline{\underline{A}}_k + G_k \underline{\underline{A}}_{k+1}$$

can be obtained only if the sequence γ_k is either independent or a stationary Markov claim with idempotent transition matrix. This latter case includes some interesting mixture processes. The results of this aspect of the project are discussed in detail in [5].

B. Adaptive Processing of Point Processes

Optical radar returns can be considered as randomly-arriving, random-amplitude pulses and point processes appear to be a natural way to model these pulses which are basically discontinuous in time. The interarrival times have been modeled with a two-parameter Gamma distribution. Letting x denote the interarrival time, the density is

$$f(x|\mu,k) = \frac{\mu^k}{\Gamma(k)} x^{k-1} e^{-\mu x}$$

With two free parameters, k and μ , this distribution can model a variety of returns. Although the interarrival times are independent, the counts are not, i.e., they are not necessarily Poisson. This would be the case, for example, with returns from an extended target such as a submarine.

It is unrealistic to assume precise knowledge of the two parameters μ and k. Assigning a prior density, $\pi(\mu,k)$, one can then define a marginal density

$$f(x) = \iint f(x|\mu,k)\pi(\mu,k)dkd\mu$$

and use this for likelihood processing. Let.

$$\theta_1 = -k/\mu$$
, $\theta_2 = k$

and define the sufficient statistic

$$\underline{t} = (t_1, t_2)'$$

$$t_1 = \sum_{i=1}^{n} x_i$$
, $t_2 = \sum_{i=1}^{n} \ln(x_i)$

In Ref. [6], it is shown that the likelihood ratio exhibits a two-dimensional estimator-correlator structure:

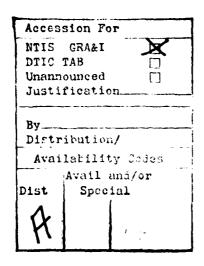
$$1(\underline{t}) = \int_{\underline{t}_0}^{\mathbf{A}} [\theta_1(u_1, u_2)du_1 + \theta_2(u_1, u_2)du_2]$$

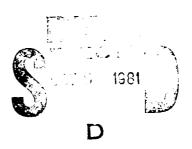
This quantity is then compared to a threshold which contains parameters from the null (no target) hypothesis. The θ_i are conditional mean estimates, i.e.,

$$\hat{\theta}_{i} \triangleq E(\theta_{i} | t_{1}, t_{2}).$$

We have studied adaptive procedures since accurately modeling optical returns is usually very difficult and one must learn the channel characteristics. This observation is summarized by saying we do not know precisely the prior pdf $\pi(\mu,k)$ and, hence, cannot compute the estimates θ_i . By adaptive, then, we mean approximating the conditional mean estimate by other estimates requiring less knowledge. These estimates are then used in the detector structure in place of the θ_i . In Ref. [6], a number of suboptimum procedures have been evaluated. These include: the MLE detector; the truncated MLE detector in which just the boundaries (range) of the parameters μ and k are assumed known; the discrete MLE where k is assumed to take on one of a discrete set of values.

The simulation results ([6], page 18) indicate that performance is surprisingly good when compared to the performance of the optimum detector. This is true even for the important small sample case - an especially encouraging result.





C. References

- McAuly, A., "Computerized Model Demonstrating Magnetic Submarine Localization," <u>IEEE Trans. Aerosp. and</u> <u>Electronic Systems</u>, AES-13, No. 3, pp. 246-254, May 1977.
- 2. Robertazzi, T. G. and Schwartz, S. C., "The Off-Line Use of a Sequential Estimator," O.N.R. Tech. Dept. 4, ISS Laboratory, Princeton University, 1979.
- 3. Tenney, R.R., Hebbert, R.S., and Sandell, N.R., "A tracking filter for maneuvering sources," <u>IEEE Trans.</u> <u>Auto. Contr.</u>, Vol. 22, pp. 246-251, April 1977.
- 4. Hadidi, M.T. and Schwartz, S. C., "Sequential Detection with Markov Interrupted Observations" Proc., Sixteenth Annual Allerton Conference on Communication, Control and Computing, October 4-6, 1978, pp. 79-88. Also issued as O.N.R. Tech. Rept. 3, Information Sciences and Systems Laboratory, Princeton University, 47 pages.
- 5. Hadidi, M. and Schwartz, S. C., "Linear Recursive State Estimators under Uncertain Observations" <u>IEEE Trans. on Auto. Control</u>, Vol. 24, December 1979, No. 6, pp. 944-948. Also issued as O.N.R. Tech. Rept. 2, Information Sciences and Systems Laboratory, Princeton University, September 1978, 24 pages.
- 6. Fogel, A., and Schwartz, S. C., "Adaptive Detection of Renewal Processes," 1978 Conf. on Information Sciences and Systems, The John Hopkins University, March 29-31, 1978, pp. 139-144. Also issued as O.N.R. Tech. Rept. 1, Information Sciences and Systems Laboratory, Princeton University, Princeton, New Jersey, September 1978, 42 pages.
- 7. Masreliez, C.J., "Approximate Non-Gaussian Filtering with Linear State Observation Relations," <u>IEEE Trans. on Auto. Control</u>, Vol. AC-20, pp. 107-111, Feb. 1975.
- 8. Morris, J., "The Kalman Filter: A Robust Estimator for some Classes of Linear Quadratic Problems," <u>IEEE Trans. Inform. Theory</u>, Vol. IT-22, pp. 526-534, Sept. 1976.
- Middleton, D. and Esposito, R., "Simultaneous Optimum Detection and Estimation of Signals in Noise," <u>IEEE Trans</u>. <u>Inform. Theory</u>, Vol. IT-14, pp. 434-444, May 1968.

- Prasad, R. M., and Sinha, A.K., "On Bootstrap Identification Using Stochastic Approximation," <u>IEEE Trans. Auto Control</u>, Vol. 22, pp. 671-672, August 1977.
- 11. Pandya, R.N., "A Class of Bootstrap Estimators and Their Relationship to the Generalized Two Stage Least Squares Estimators," <u>IEEE Trans. on Auto Control</u>, Vol. AC-19, Dec. 1974, pp. 831-835.
- 12. Golub, G., and Pereyra, V., "The Differentiation of Pseudo-Inverses and Nonlinear Least Squares Problems Whose Variables Separate," <u>SIAM J. Numer. Anal.</u>, Vol. 10, April 1973, pp. 413-432.
- 13. Middleton, D., "Probability Models of Received Scattered and Ambient Fields," <u>Proc. 6th Annual Princeton Conf. on Information Sciences and Systems</u>, Princeton, New Jersey pp. 66-71, March 1972.
- 14. Parzen, E., Stochastic Processes, Holden Day, 1962

D. Reports, Publications, Thesis

- A. Fogel and S. C. Schwartz, "Adaptive Detection of Renewal Processes," 1978 Conf. on Information Sciences and Systems, The John Hopkins University, March 29-31, 1978, pp. 139-144. Also issued as O.N.R. Tech. Rept. 1, Information Sciences and Systems Laboratory, Princeton University, Princeton, New Jersey, September 1978, 42 pages.
- M. Hadidi and S. C. Schwartz, Linear Recursive State
 Estimators under Uncertain Observations" <u>IEEE Trans. on</u>
 <u>Auto Control</u>, Vol. 24, December 1979, No. 6, pp. 944-948.
 Also issued as O.N.R. Tech. Rept. 2, Information Sciences and Systems Laboratory, Princeton University, September 1978, 24 pages.
- 3. M. T. Hadidi and S. C. Schwartz, "Sequential Detection with Markov Interrupted Observations" Proc., Sixteenth Annual Allerton Conference on Communication, Control and Computing, October 4-6, 1978, pp. 79-88. Also issued as O.N.R. Tech. Rept. 3, Information Sciences and Systems Laboratory, Princeton University, 47 pages.
- 4. M. T. Hadidi, Estimation and Detection for Observation Models with Switching, Ph.D. Thesis, July 1979.
- T. G. Robertazzi and S. C. Schwartz, "The Off-Line Use of a Sequential Estimator," In preparation (approx. 78 pages).

E. Technical Meetings, Seminars and Talks

- 1. Item 1 under publications was presented at the Johns Hopkins Conference on Information Sciences and Systems, March 29-31, 1978, Baltimore, Maryland.
- "On Robust Detection Techniques" Presentation at ONR/NCSC Workshop on Robustness in Sonar Systems, November 20-21, 1978, Panama City, Florida.
- 3. Item 3 under publications was presented at the Allerton Conference on Communication, Control, and Computing, October 4-6, 1978, Urbana, Illinois.
- 4. Organized and chaired a session on "Adaptive Techniques in Communications" at the Communications Theory Workshop, April 23-25, 1979, Trabuco Canyon, California.

- 5. Seminar, "Adaptive Detection of Renewal Processes,"
 Statistics Dept., Princeton University, Nov. 16, 1978;
 also presented at Dept. of Systems Engineering, Moore
 School of Electrical Engineering, University of
 Pennsylvania, Dec. 11, 1978.
- 6. Seminar, "Estimator-Correlator Receivers," Electrical Engineering Dept., University of Texas, Austin, Texas, February 22, 1980.

F. Personnel Supported by contract

<u>Principal Investigator</u> Stuart Schwartz

50% time, 2 months, summer of 1978, 1979.

Assistants in Research

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100%, Feb. 1978 - July 1978 100%, Sept.1977 - June 1978 100%, Feb. 1978 - July 1979 100%, Sept.1978 - Dec. 1979

